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Weddings with Uncertain Prospects — Mergers under Asymmetric Information

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ABSTRACT: We provide a framework for analyzing bilateral mergers when there is two-sided asymmetric information about firms' types. We show that there is always a "no-merger" equilibrium where firms do not consent to a merger, irrespective of their type. There may also be a "cut-off" equilibrium if the expected merger returns satisfy a suitable single crossing condition, which will hold if a firm's merger returns are "essentially monotone decreasing" in its type. Applying our analysis to the linear Cournot model, we show how the merger pattern depends on the cost effects of mergers, the extent of uncertainty, and the way profits are split. Specifically, we show how increasing uncertainty about competitor types may foster mergers as firms hope for strong rationalization effects.

Keywords: merger, asymmetric information, oligopoly, single crossing.

JEL: D43, D82, L13, L33.

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1 Introduction

When Dynegy was considering a takeover of Enron in autumn 2001, it was certainly aware that the potential acquiree was in trouble. Quite clearly, however, Dynegy initially underestimated the magnitude of the problem.[1] After the company had found out more about its target, it invoked a "material adverse change" clause to retreat from the deal. Yet, not all firms that recently considered mergers or acquisitions were lucky enough to find ways out of a transaction with partners that turned out to be less attractive than expected. For example, in the merger with the German Hypobank, it took Hypovereinsbank more than two years "to discover the full horror of its partner's balance sheet" (*The Economist*, July 20, 2000). More generally, many mergers are considered as failures with the benefit of hindsight.[2]

Such anecdotes suggest that standard results from the analysis of "lemons" markets (Akerlof 1970) can explain the mechanics of mergers under asymmetric information. However, there are two reasons why this is not clear. First, even though anecdotes on failed mergers and takeovers typically single out one of the partners as the lemon, the asymmetric information surrounding mergers is usually two-sided: Each of the firms knows more about its quality than the potential partner does. Both parties thus face the risk of joining a bad partner who adversely affects the profits of the merged entity. In the present paper, we therefore consider mergers under *two-sided asymmetric information* about firms' types.[3] High types are defined as having high

[1] Dynegy's former CEO C. Watson is cited in *The Economist* (November 15, 2001) to have "looked under the hood", finding that Enron "might need a new paint job and some new tyres, but its engine is sound."

[2] See, for instance, the studies of Ravenscraft and Scherer (1987; 1989), but note also the results of Healy et al. (1992).

[3] Hviid and Prendergast (1993) provide an analysis of merger games with one-sided asymmetric information. Assuming that the target firm has private information about its profitability, they show that an unsuccessful bid may increase the profitability of the target but reduce the profitability of the bidding firm (relative to the profitability before the merger offer) due to learning from rejection.

stand-alone profits—that is, high profits in the absence of a merger—and as contributing to high merger profits if the transaction occurs.

Second, while the market for firms resembles a standard lemons market in the sense that low types have lower stand-alone profits (*stand-alone profit effect*), there is a crucial difference: In contrast to the lemons market, lower types will generally also have lower profits than high types if a transaction occurs, since low types will drive down profits of the merged entity. Assuming that lower profits of the merged entity translate into lower profits for the former owners of each of its constituent parts, low types will expect to earn less than high types if they become part of a merged firm (*post-merger profit effect*).[4] This feature distinguishes mergers from transactions like the sale of used cars, where the seller's profit is independent of the type of the car, as all types must sell at the same price.[5] As a result, it is not obvious whether low types have more to gain from entering a merger than high types.

Against this background, we set out to analyze issues that are familiar from the adverse selection literature. Specifically, we investigate under which conditions asymmetric information about the firms' types might lead to a breakdown of the market for firms, that is, to a no-merger equilibrium where firms never consent to a transaction. Moreover, we establish conditions under which, in equilibrium, only the relatively low types are going to merge, as the lemons analogy would suggest.

To this end, we analyze a merger game in which two firms are matched whose types $z_i, i = 1, 2$, are drawn from distributions that are common knowledge.[6] After having observed their own type, both firms state whether they consent to a merger. If both firms consent, a merger takes place. If at

[4] When the transaction is financed with cash, the post-merger profit effect is absent for the owners of the acquired firm. Even for them, however, knowledge of the competitor's type would be helpful for the acquisition decision, as stand-alone profits and thus merger returns depend on this type.

[5] We are obviously abstracting from warranty payments here.

[6] In specific applications, types may be interpreted as cost or demand parameters, with lower cost or higher demand corresponding to a better type.

least one firm declines, there is no merger. Following the merger game, an oligopoly game is played. If no merger occurs, both firms earn their stand-alone profits. If a merger occurs, the joint profit is shared according to some predetermined rule.[7]

We develop our theory in a relatively general reduced-form model and illustrate its application in a set of examples based on the standard linear Cournot oligopoly model. For the general model, we first show that the merger game always has a Bayesian equilibrium where players never consent to a merger, no matter what their type is. The remaining results depend on the relation between firm types and merger returns, which are defined as the difference between the pre- and post-merger profits for the owners of each firm. For instance, we give conditions on this relation guaranteeing that the no-merger equilibrium is unique—corresponding to a "no-trade" result.

Next, we analyze the merger pattern, i.e., we ask what types of firms consent to mergers under two-sided asymmetric information. Unsurprisingly, when the merger returns are decreasing in the type of a firm, the equilibria will be of the cut-off type where only low types consent to a merger. This result is potentially relevant to the "Merger Puzzle" (Scherer 2002), that, in spite of their ubiquity, mergers are often considered as failures: If bad types are more likely to merge than good types (adverse selection), this would suggest that merged entities do badly because they consist of bad firms.

The cut-off result for monotone merger returns is useful in our applications, but it has some limitations: First, as argued before, the monotonicity of the merger returns is not guaranteed in general. Therefore, we show that the cut-off result generalizes to the class of "essentially monotone decreasing

[7] We realize that our model is highly reduced. Obviously, the terms of a merger are usually the outcome of a complex bargaining process. However, the literature on bargaining with two-sided incomplete information (e.g. Ausubel et al. 2002) shows that such models often admit a multitude of equilibria, and the outcomes often depend delicately on the details of the bargaining protocol. Therefore, we believe that the reduced form approach adopted in this paper is useful for making progress with respect to the issues we are interested in.

functions", which contains many single-peaked functions. Second, the cut-off result does not preclude degenerate cases where either all types consent to a merger or only the "no-trade" equilibrium exists.[8] We thus provide both necessary and sufficient conditions for non-degenerate cut-off equilibria to exist.

We then apply our results in a linear Cournot setting. We show how merger patterns depend on the following underlying conditions:

(i) The technology of the merged firm and, specifically, the effect of the merger on the marginal cost of the merged firm;

(ii) The profit sharing rule, that is, the mechanism determining how post-merger profits are split between the owners of the two constituent firms;

(iii) The extent of uncertainty surrounding the transaction.

We start with the case of *rationalization mergers*, where the merged firm produces with the minimum of the marginal costs of its constituent parts. Under symmetric information, such mergers increase joint profits if the firms are sufficiently heterogenous because the more efficient firm rationalizes the less efficient competitor (Barros 1998), whereas this is impossible for homogeneous firms (Salant et al. 1983). With this in mind, we prove the following results. First, if firms have to commit to a fixed split of profits before a merger takes place, then there can be no symmetric equilibrium where a positive measure of firms consents to a merger under uncertainty. Second, when firms commit to fixed shares of the joint surplus or pay cash for the transaction, mergers will only take place when the uncertainty is sufficiently large. Intuitively, when uncertainty is large, firms can hope that they will turn out to be sufficiently asymmetric ex post for rationalization effects in the sense of Barros to materialize, whereas for small uncertainty they know they will essentially be in the Salant et al. case ex post.

[8] In these degenerate cases, the cut-off value are identical to the highest or the lowest possible type, respectively.

Finally, we compare the results for rationalization mergers with those for *synergy mergers*, where the merged entity produces with the best available technology. Here, we focus on the case where firms commit to a fixed split of profits. In this setting, we generally obtain cut-off equilibria.

The remainder of the paper is organized as follows. In section 2, we introduce the main assumptions of our model. Section 3 characterizes the Bayesian equilibria for the class of games under consideration, primarily focussing on the case where merger returns are monotone decreasing in the firms' types. Section 4 provides a detailed analysis of horizontal mergers in the linear Cournot setting when there is two-sided asymmetric information. In section 5, we generalize our analysis to cases where merger returns are not monotone decreasing in firm types. Section 6 concludes.

2 Assumptions

We consider an oligopoly with an exogenous number of firms $n \geq 2$. Two of these firms, denoted as $i = 1, 2$, play a merger game. The firms may be active in the same market (and thus contemplate a horizontal merger). Alternatively, they might be operating in a vertical relationship (vertical merger), or producing unrelated goods (conglomerate merger). Each firm is characterized by a type $z_i \in \mathbb{R}$, which influences its profitability. There is asymmetric information about the value of z_i, i.e. firms know their own z_i, but not their competitor's $z_j, j \neq i$. The ex ante probability of z_i is described by a probability distribution F_i with density f_i and compact support $[\underline{z}_i, \overline{z}_i] \subset \mathbb{R}$.[9] F_i is common knowledge. Note that we allow for ex ante heterogeneity between firms, i.e. firms' types z_i may be drawn from different distributions.[10]

[9] It is possible to extend the analysis to the case where the support of F_i is not compact, though we do not pursue this issue here.

[10] This is of particular importance for vertical or conglomerate mergers where firms are producing entirely different goods. Even the interpretation of the firms' types might differ. For vertical mergers, for instance, the types might correspond to the costs of input production for the upstream firm and marketing ability for the downstream firm.

Firms simultaneously announce whether they are willing to merge.[11] The decision of firm i is summarized in a variable s_i such that $s_i = 1$ if it consents to an agreement and $s_i = 0$ if it rejects it. If no merger occurs, each firm earns its stand-alone oligopoly profit $\pi_i(z_i, z_j)$. This function is defined on some set $\mathcal{Z} \supset [\underline{z}_1, \overline{z}_1] \times [\underline{z}_2, \overline{z}_2]$. The properties of π_i reflect more primitive assumptions on the nature of product market interaction and the interpretation of the type variable. At this stage, we do not want to constrain the shape of $\pi_i(z_i, z_j)$ too much. However, throughout the paper, we shall require the following assumption to be satisfied.

Assumption 1 *π_i is non-decreasing in z_i.*

Thus, by definition, the higher a firm's type, the higher its stand-alone profits. With respect to the relation between z_j and $\pi_i, j \neq i$, we make no assumption at this stage to allow for various forms of firm interaction. For instance, if firms are competitors and the type variable reflects efficiency, greater efficiency of firm j translates into lower profits of firm i in most applications. However, if firms i and j are in a vertical relation, the opposite relation is more plausible.

If a merger occurs, the merged entity earns total profit $\pi^M(z_i, z_j)$. This profit is shared between the owners of each of the formerly separate firms. The owners of firm i earn profits $\pi_i^M(z_i, z_j), i, j = 1, 2, j \neq i$, such that

$$\pi_1^M(z_1, z_2) + \pi_2^M(z_2, z_1) = \pi^M(z_1, z_2).$$

Like π_i, the functions π_i^M and π^M reflect assumptions on product market interaction and the interpretation of the type variable. In addition, π_i^M depends on the way profits are shared if a merger takes place. There is no commonly accepted theory of how profits of a merged entity are split between the owners of formerly separate firms. We shall therefore not impose much structure on the general model. It is natural to suppose that the split

[11] Sequentiality of decisions does not lead to substantial changes of the results (see Borek et al. 2003).

of profits reflected in π_i^M is the outcome of a bargaining process that precedes the merger decision summarized in s_i. In this bargaining process, firms typically reveal information about their types and they are thus able to update their beliefs about the potential partner's type. That is, the distribution functions $F_i, i = 1, 2$, describing the probability of z_i should be regarded as being conditional on any information revealed in the bargaining process. For our analysis to be interesting, we require that, whatever the bargaining process, firms do not fully reveal their types before final acceptance decisions are made.

We maintain the following assumption on π^M.

Assumption 2 π^M *is non-decreasing in* z_i *and* z_j.

This assumption is natural: The more efficient the constituent parts, the more efficient should the merged firm be. Note that we do not make any assumptions on the effect of the type variable on π_i^M, thereby avoiding assumptions on the bargaining process. Instead, we shall show how different ways of splitting the pie will translate into different predictions on the merger pattern. We shall consider the following three profit sharing rules.

Fixed Profit Shares *Firm* i *obtains a predetermined share* $\alpha_i \in [0, 1]$ *of the merged entity's total profit* $\pi^M(z_i, z_j)$, *i.e.*

$$\pi_i^M(z_i, z_j) = \alpha_i \pi^M(z_i, z_j).$$

The Fixed Profit Shares rule imposes that firms ex ante commit to a particular split of profits, even if one firm turns out to be very inefficient ex post. This is essentially the way profits are shared if the owners of a merging firm are compensated for bringing in assets by shares in the new firm.

Joint Surplus Sharing *Firm* i *obtains its stand-alone profit plus a predetermined share* $\beta_i(z_i, z_j) \in [0, 1]$ *of the total change in profits, i.e.*

$$\pi_i^M(z_i, z_j) = \pi_i(z_i, z_j) + \beta_i(z_i, z_j)\left[\pi^M(z_i, z_j) - \pi_i(z_i, z_j) - \pi_j(z_j, z_i)\right].$$

According to the Joint Surplus Sharing rule, the owners agree on a contract such that both win if total profits increase and both lose if total profits fall. We do not claim that the joint surplus rule corresponds to a common real-world case. However, such a procedure is conceivable when types are revealed (and verifiable) after the merger takes place, so that contracts can be conditioned on types.

Cash Payment *The owners of one firm, say firm 2, are compensated by the cash payment $p > 0$ for the takeover by the other firm, i.e.*

$$\begin{aligned} \pi_1^M(z_1, z_2) &= \pi^M(z_1, z_2) - p; \\ \pi_2^M(z_2, z_1) &= p. \end{aligned}$$

The Cash Payment rule brings the setting closer to one-sided asymmetric information. Even though neither firm knows the competitor's type, the post-merger profit for firm 2's owners is independent of firm 1's type: They are compensated with a fixed cash payment. Nevertheless, the asymmetric information remains relevant for firm 2's merger decision, as the type of firm 1 influences firm 2's stand-alone profits and thus its merger returns.

Henceforth, we shall use the following formal definition of merger returns:

Definition 1 *Firm i's* ***returns from the merger*** *are given by*

$$g_i(z_i, z_j) \equiv \pi_i^M(z_i, z_j) - \pi_i(z_i, z_j).$$

The form of the function g_i will determine our main results. Note that Assumptions 1 and 2 impose little structure on g_i. In fact, it is not even clear that merger returns are decreasing in own type, that is, higher types are less likely to enter merger agreements. By Assumption 1, better types would earn higher stand-alone profits. By Assumption 2, however, they would also earn higher profits in a merged firm. The net effect is unclear.

3 Results

In this section, we characterize the Bayesian equilibrium of the class of merger games under consideration in general terms.[12] The results in this section are chosen because they are straightforward to apply to particular examples (see section 4). Section 5 contains more general results, which, however, are not as simple.

The following notation is useful: For $i = 1, 2$, if firm i plays a strategy $s_i(z_i)$, we define $B_i \equiv B_i(s_i) \equiv \{z_i \,|s_i(z_i) = 1\}$, i.e., B_i denotes the set of types z_i for which firm i consents to a merger. Further, let

$$G_i(z_i; B_j, f_j) \equiv \int_{B_j} g_i(z_i, z_j) f_j(z_j) \, dz_j$$

denote the expected merger returns for firm i with type z_i when players j are distributed as f_j, and only players in B_j consent to a merger.

3.1 Cut-Off Equilibria

First, we give conditions under which low types are more likely to merge in equilibrium, that is, there is a cut-off equilibrium where only low types consent to a merger. Therefore, for cut-off equilibria there is a monotone relation between types z_i and strategies s_i. Such equilibria are common in Bayesian games: Examples include first-price auctions where the type is the bidder's valuation and the strategy is the bid, double auctions where the types of buyers and seller are valuations and costs, and the strategies are bids and asks (Chatterjee and Samuelson 1983), wars of attrition where the type is the valuation for the prize and the strategy is the quitting period, and games of public good provision where types correspond to the costs of providing a public good and actions correspond to the provision decision.[13] Athey (2001) analyzes more generally under which such monotone equilibria arise.

[12] We shall apply our general analysis in a linear Cournot setting in Section 4.

[13] See Fudenberg and Tirole (1991) for a discussion of these games.

We shall use the following terminology.

Definition 2 *The function* $G_i : [\underline{z}_i, \overline{z}_i] \rightarrow \mathbb{R}$ *satisfies* ***strong downward single crossing (SSC⁻)*** *if, for all* $z_i^H, z_i^L \in [\underline{z}_i, \overline{z}_i]$ *such that* $z_i^H > z_i^L$, $G_i\left(z_i^H\right) \geq 0$ *implies* $G_i\left(z_i^L\right) \geq 0$ *and* $G_i\left(z_i^H\right) > 0$ *implies* $G_i\left(z_i^L\right) > 0$.

This definition is closely related to the familiar single-crossing property of incremental returns (Milgrom and Shannon 1994).[14] We first give a cut-off condition in terms of expected merger returns, and then consider more primitive conditions on actual merger returns g_i.[15]

Lemma 1 *Suppose* $G_i\left(z_i; B_j, f_j\right)$ *satisfies* SSC^- *in* z_i *for all* $B_j \subset \mathcal{Z}_j$ *and all* f_j. *Then every Bayesian Equilibrium* (s_1^*, s_2^*) *in pure strategies with* $\mathbb{P}\left[B_i\left(s_i^*\right)\right] > 0$ *satisfies the* ***cut-off-property****, that is, there are cut-off values* $z_i^* \in \mathcal{Z}_i$ *such that*

$$s_i^*\left(z_i\right) = \begin{cases} 1, & \text{if } z_i \leq z_i^*; \\ 0, & \text{if } z_i > z_i^*; \end{cases} \qquad i = 1, 2.$$

Proof. See Appendix. ■

The intuition is straightforward. SSC^- states that, for any distribution of z_j, if some type z_i consents to a merger, so will any lower type $z_i' < z_i$, no matter what the distribution of z_j is. The result applies this property to the distribution of z_j corresponding to the equilibrium behavior of z_j.

Lemma 1 immediately implies the following result.

Proposition 1 *If* $g_i\left(z_i, z_j\right)$ *is monotone decreasing in* z_i*, then every Bayesian Equilibrium satisfies the cut-off property.*

[14] Let $\Pi_i\left(s_i, z_i; B_j, f_j\right)$ define the expected payoff from strategy s_i for a firm with type z_i, facing a competitor characterized by B_j and f_j. Then $\Pi_i\left(s_i, z_i; B_j, f_j\right)$ satisfies the Milgrom-Shannon Single-Crossing Property in $\left(-s_i, z_i\right)$ if and only if G_i satisfies SSC^-.

[15] Using the equivalence between SSC^- and the Milgrom-Shannon condition, Lemma 1 is a special case of Theorem 1 in Athey (2001).

The intuition for this result is simple: If higher types have less to gain from a merger for arbitrary realizations of types, then clearly they must gain less in expectation.

Proposition 1 does not exclude degenerate cut-off equilibria where all or no types are willing to merge. The following result provides a simple condition under which such non-degenerate equilibria exist.[16]

Corollary 1 *If g_i is monotone decreasing in $z_i, i = 1, 2,$ and if for all $i, j = 1, 2, i \neq j$, there exist $z_i^e \in (\underline{z}_i, \overline{z}_i)$ such that*

$$\int_{\underline{z}_j}^{z_j^e} g_i\left(z_i^e, z_j\right) f_j\left(z_j\right) dz_j = 0, \tag{1}$$

then there is a Bayesian Equilibrium (s_1^*, s_2^*) of the simultaneous merger game in pure strategies such that the cut-off values satisfy $z_i^* = z_i^e$.

The result is straightforward to prove. Intuitively, Corollary 1 states that, in equilibrium, firm i with the cut-off type $z_i^e = z_i^*$ is just indifferent between consenting to the merger and rejecting it, since the expected returns from merging with types below the cut-off level z_j^e equal zero. That is, since g_i is decreasing in z_i by assumption, all types below z_i^e will consent to the merger, whereas all types above z_i^e will reject it.

Together, Proposition 1 and Corollary 1 say that the lemons logic carries over to our setting with two-sided asymmetric information, provided that merger returns are monotone decreasing in own type. Our linear Cournot example in section 4 will illustrate, however, that the monotonicity condition is surprisingly restrictive. We shall therefore provide a generalization of the monotonicity condition to a wider class of functions, including many single-peaked functions, in section 5. Yet, even this generalized monotonicity condition turns out to be violated quite often.

[16] It is possible to derive a uniqueness condition, which makes sure that one reaction function is always steeper than the other. As this condition is not particularly illuminating, we refrained from stating it.

The cut-off nature of the equilibrium suggests that low types are more likely to merge than high types if the relevant conditions hold. This conclusion has an interesting implication that is relevant to a better understanding of the "Merger Puzzle" (Scherer 2002), that, in spite of their ubiquity, mergers often turn out to be failures: If firm types are drawn from *identical* distributions, merged entities perform badly simply because of adverse selection.

However, this conclusion needs to be interpreted carefully. For suppose there is ex ante heterogeneity, i.e., firms' types are drawn from *different* distributions. Assume for simplicity that firm 2 is chosen from a distribution that is generated by a shift of firm 1's distribution to the right. Then a low-type firm 2 with state $\tilde{z}_2$ consenting to a merger might have a higher type than a high-type firm 1 with state $\tilde{z}_1$ that does not consent to a merger. Figure 1 illustrates this argument.

<Figure 1 around here>

Another simple property of the cut-off equilibrium is that there typically is a non-degenerate measure of types for which mergers are not profitable ex post: Types that are just below the cut-off level break even in expectation, but make losses if the partner is drawn from the lower tail of the distribution ("ex post regret"). That is, mutual uncertainty about the potential partner's type may help explain why mergers often turn out to be non-profitable ex post.

This feature of the model contrasts with familiar results on two-sided asymmetric information, for instance, in the context of double auctions, where the valuations of buyers and the costs of sellers are private information (Chatterjee and Samuelson 1983). While inefficient no-trade outcomes loom large in this literature, ex post regret about trade is not an equilibrium phenomenon. This follows from the fact that knowledge of the other type matters only for how much can be extracted from the other party. In the context of mergers, however, it is also relevant for one's own valuation of the transaction. More generally, the fact that valuations are endogenous—influenced by both players' types via their market interaction—rather than exogenous (and equivalent to one player's type) complicates our analysis.[17]

3.2 No-Merger Equilibrium

We first derive a simple no-merger result: For arbitrary distributions of types, there is always a degenerate cut-off equilibrium where no type merges.

Proposition 2 (no-merger) *Each strategy pair* (s_1, s_2) *with*

$$\mathbb{P}\left[B_i\left(s_i\right)\right] \equiv \int_{B_i} f_i\left(z_i\right) dz_i = 0, \quad i = 1, 2,$$

is a Bayesian Equilibrium of the merger game.

Proof. See Appendix. ■

The result is very intuitive: If both firms believe that the other firm will not consent to a merger—no matter what its type is—it is a (weakly) best response not to consent, and beliefs are correct in equilibrium. Thus, there always is an equilibrium where firms merge with probability zero. Note, however, that the no-merger equilibrium is Pareto-dominated in terms of expected profits whenever a cut-off equilibrium exists where firms consent to a merger with strictly positive probability.

The next proposition gives conditions on the expected merger returns guaranteeing that there is no other equilibrium.

Proposition 3 *Suppose* $\mathcal{Z}_i = \mathcal{Z}_j = \mathcal{Z}$. *Further assume that, for* $i, j = 1, 2, j \neq i$, g_i *is non-increasing in* z_i *and non-decreasing in* z_j.

(i) If, for all $i \in \{1, 2\}$, *there is no* $\widehat{z}_i$ *such that* $g_i\left(\widehat{z}_i, \widehat{z}_i\right) \geq 0$, *then there is no Bayesian Equilibrium with* $\mathbb{P}\left[B_i\left(s_i^*\right)\right] > 0$.

[17] A parallel arises in the auctions literature: Jehiel and Moldovanu (2000) have recently considered second-price, sealed-bid auctions where the valuation for the object is also determined endogenously through the market interaction of players.

(ii) If, for at least one $i \in \{1, 2\}$, there is no $\widehat{z}_i$ such that $g_i(\widehat{z}_i, \widehat{z}_i) \geq 0$, then there is no symmetric Bayesian Equilibrium with identical cut-off values where $P[B_i(s_i^*)] > 0$.

Proof. See Appendix. ■

Thus, even though the Assumptions of Proposition 3 are consistent with Proposition 1, so that every equilibrium must be of the cut-off type, only the degenerate cut-off equilibrium with $P[B_i(s_i^*)] = 0$ can exist.

The result is useful in applications such as the linear Cournot model with $n \geq 3$ firms where, for homogeneous firms, joint profits decrease with a merger (see section 4). Then, $g_i(\widehat{z}_i, \widehat{z}_i) < 0$ must hold for at least one firm. It follows immediately that there cannot be a symmetric equilibrium. If, in addition, profits are shared such that $g_i(\widehat{z}_i, \widehat{z}_i) < 0$ for both firms and arbitrary $\widehat{z}_i$, then at least one firm must have negative merger returns for every conceivable combination of types, and thus negative expected merger returns. As a result, there is no equilibrium where firms merge with strictly positive probability.

4 Example: Horizontal Cournot Model

We now apply our results to horizontal mergers in a linear Cournot setting, where we think of the type as the negative of marginal costs. We shall show how predictions on the merger pattern depend on (i) the effect of the merger on the new entity's marginal costs, (ii) the sharing rule adopted by the merging firms, and (iii) the extent of asymmetric information.

As a background, recall the following results familiar from the literature on mergers in the Cournot model without asymmetric information. Salant et al. (1983) have shown that bilateral mergers of homogeneous firms are never profitable (except for mergers to monopoly) if they do not reduce marginal costs. Barros (1998) notes that bilateral mergers may be profitable when firms are heterogenous: If the merged entity inherits the technology of the more efficient firm, and the marginal costs of the merging firms differ sufficiently, a rationalization effect will render the merger profitable.

Now consider a setting with two-sided asymmetric information. Suppose there are three firms with marginal costs $c_i, i = 1, 2, 3$, and inverse demand is given by $P(Q) = a - bQ$, where $Q = \sum_i q_i$ is aggregate output and $a, b > 0$. We consider a merger game between firm 1 and 2. The firms' types are defined as $z_i \equiv -c_i$, i.e. the negative of marginal costs. Suppose that z_1 and z_2 are uniformly distributed with compact support $\mathcal{Z} = \mathcal{Z}_1 = \mathcal{Z}_2 = [\underline{z}, \overline{z}]$. As in Barros (1998), we first suppose that the merged firm inherits the technology of the more efficient firm (*rationalization mergers*). The type of the merged firm is thus given by $z^m = \max(z_1, z_2)$. We then consider mergers that reduce the level of marginal cost even below that of the more efficient firm, i.e. mergers that give rise to synergies (*synergy mergers*). More precisely, in the synergy case, we suppose that the merged firm produces with the lowest marginal cost that any firm could conceivably have, i.e., $z^m = \overline{z}$. Thus, the synergy case correspond to an extremely favorable view of the merger effects.

We shall use the following specific parameter values: $a = 200, b = 1, c_3 = 100$. Furthermore, we use $\gamma \in [0, 20]$ to define the support of the type distribution as

$$\mathcal{Z} = [-(100 + \gamma), -(100 - \gamma)],$$

i.e., an increase in γ amounts to a mean-preserving spread (see e.g. Laffont 1983, pp. 24). We have chosen γ so that for all possible combinations of types, all firms produce positive outputs. In Figure 2, the area where all firms produce positive output is given by ABC. The shaded areas ABE and BCD, respectively, indicate marginal cost combinations that generate increases in total profits in case of a merger.[18] The hatched area BDE gives the cost combinations for which a merger reduces the total profits of the merging firms.

<Figure 2 around here>

[18]For cost combinations in ABE, firm 1 rationalizes firm 2. For cost combinations in BCD, the rationalization effect is reversed.

Our analysis of the equilibrium behavior distinguishes:

(i) rationalization and synergy mergers,

(ii) three sharing rules (Fixed Profit Shares, Joint Surplus Sharing, and Cash Payment),

(iii) three different distributions of types ($\gamma = 2, 5$ and 20, respectively).

In the following, we emphasize the most interesting results that emerge from the Cournot model, and we sketch why these results hold.[19]

4.1 Rationalization Mergers

Table 1 gives a brief description of the equilibrium structure for the case of rationalization mergers, indicating which of the following cases arises:

1) Only the no-merger equilibrium exists ($\emptyset$ or $\emptyset^*$).

2) There is an equilibrium where all firms consent to the merger.

3) There is a non-degenerate cut-off equilibrium.

In case 1), the starred entries have to be taken with a grain of salt: In those cases, we only considered "simple" equilibria where the set of players that consent to a merger is an interval. We have not excluded the possibility that more complicated equilibria could arise.

<Table 1 around here>

[19] Details of calculations are available on request from the authors.

4.1.1 Fixed Profit Shares

Recall from section 2 that under the Fixed Profits Shares rule, firm $i = 1, 2$ gets a predetermined share of the merged entity's total profits. In this case, we obtain the result that uncertainty does not matter: As for homogeneous firms under certainty, there are no mergers under asymmetric information and firms that are ex-ante homogeneous.

Observation 1 *For rationalization mergers with fixed profit shares* $\alpha_i \in [5/32, 27/32]$ *and arbitrary* $\gamma \in [0, 20]$, *there is no symmetric Bayesian equilibrium where firms 1 and 2 merge with positive probability.*

This observation follows from Proposition 3(ii). To see this, suppose that firms commit to profit shares $\alpha_1, \alpha_2 \in [5/32, 27/32]$ and $\alpha_1 + \alpha_2 = 1$. For these profit shares, firm i's merger returns are monotone decreasing in own type $(\partial g_i/\partial z_i < 0)$ and monotone increasing in the competitor's type $(\partial g_i/\partial z_j > 0)$: Intuitively, as long as one firm does not obtain a very high profit share, the effects of types on merger returns are influenced more by the effects on stand-alone profits than by the effects on post-merger profits. Furthermore, by Salant et al. (1983), there must be at least one $i \in \{1, 2\}$ such that $g_i(\widehat{z}, \widehat{z}) < 0$ for any $\widehat{z} \in \mathcal{Z}$. Proposition 3(ii) implies that under these conditions there cannot be a symmetric Bayesian equilibrium, irrespective of the amount of uncertainty.[20]

It is important to note that the absence of mergers is not necessarily inefficient: As mergers reduce joint profits for sufficiently homogeneous firms, mergers are never efficient for $\gamma = 2$. For larger values of γ, the absence of mergers may well be inefficient, as potential rationalization effects will not materialize. We will now show that other sharing rules might allow for the exploitation of such rationalization effects.

[20] Furthermore, if profit shares are not too asymmetric, such that $g_i(\widehat{z}, \widehat{z}) < 0$ for all $i \in \{1, 2\}$, it follows from Proposition 3(i) that an asymmetric equilibrium where firms 1 and 2 merge with posititve probability cannot exist either.

4.1.2 Joint Surplus Sharing

Under the Joint Surplus Sharing rule, firm $i = 1, 2$ gets its former profit plus a predetermined and constant share β_i of the total change in profits. Our results are summarized in the following observation.

Observation 2 *For rationalization mergers with joint surplus sharing and low uncertainty* $(\gamma = 2)$, *only the no-merger equilibrium exists. As the uncertainty* γ *increases, non-trivial equilibria also exist where firms consent with strictly positive probability. For high uncertainty* $(\gamma = 20)$, *there is an equilibrium where all types consent to a merger.*

The intuition reflects familiar results from the case *without* uncertainty: When the uncertainty about the competitor is small, firms know they will turn out to be fairly similar ex post—too similar for a sufficient rationalization effect to materialize (as in the Salant et al. case). When uncertainty is large, there is a chance that firms turn out to be so heterogenous that the rationalization effect is sufficient to make the merger profitable (as in the Barros case).

Importantly, in the joint surplus case, merger returns are not monotone in a firm's type. This is best seen from Figure 2: For homogeneous firms, merger returns are negative. Both to the right and to the left of the diagonal, merger returns are eventually positive, as the shaded area is reached. Again, this reflects the intuition that merger returns are positive when substantial rationalization is possible, that is, when differences between firms are large enough. Because of this non-monotonicity of merger returns, cut-off equilibria are not guaranteed by Proposition 1 in this case.

4.1.3 Cash Payment

Under the Cash Payment rule, the owners of firm 2 are compensated by a cash payment $p > 0$ by the owners of firm 1. Again, we consider different amounts of uncertainty. Our results are summarized as follows.

Observation 3 *For rationalization mergers with cash payment, low or medium uncertainty ($\gamma = 2$ or $\gamma = 5$) and arbitrary price levels, only the no-merger equilibrium exists. For high uncertainty ($\gamma = 20$), a non-trivial cut-off equilibrium exists for suitable prices.*

The intuition for the cases $\gamma = 2$ and $\gamma = 5$ is similar as in the joint surplus case. A necessary condition for prices to exist such that some firms consent to a merger is that expected post-merger profits are higher than the sum of expected stand-alone profits. When uncertainty is small, firms are too similar for this to happen. For $\gamma = 20$, there exists a set of types (z_S^*, z_B^*) for sellers and buyers, such that the expected post-merger profits are higher than the sum of expected stand-alone profits before the merger. On the boundary of this set, total expected stand-alone and post-merger profits are identical and there must therefore exist a price such that both the seller and the buyer are indifferent towards the merger. Thus, the boundary of this set contains combinations of types that are cut-off values for suitable prices.

Again, merger returns are generally non-monotone in this case, so that Proposition 1 does not apply. Nevertheless, we see that a non-degenerate cut-off equilibrium may exist for suitable values of prices and uncertainty.

4.2 Synergy Mergers

Suppose now that the merger gives rise to synergies. More specifically, assume that the merged firm is of the highest conceivable type, i.e. $z^m = \overline{z}$. We confine ourselves to a discussion of the case with fixed profit shares. Our main result is the following.

Observation 4 *For synergy mergers with fixed profit shares, a cut-off equilibrium arises for $\gamma = \{2, 5, 20\}$.*

Intuitively, for synergy mergers with fixed profit shares, the post-merger profit is independent of firm types. Under these conditions, the merger return function must be monotone decreasing in own type by Assumption 2. Thus, high types always gain less from a merger than low types, which leads to cut-off equilibria. Unlike the case of rationalization mergers, transactions come about even for low uncertainty.[21] This follows from the fact that even symmetric firms may gain from mergers when there are synergies.

5 Generalizations

In this section, we consider two modifications of our analysis in section 3. First, we show how to apply our theory when the return functions g_i are monotone increasing rather than decreasing in own type. Such a setting is conceivable when the positive effects of own type on the merged entity are greater than the positive effects on stand-alone firms. As we will sketch below, a natural example arises when the type variable is relation-specific, that is, corresponds to a firm characteristic which is valuable only if the firms join forces. Second, we introduce the concept of "essentially monotone decreasing" functions and show that Proposition 1 generalizes to the case where the return functions g_i satisfy this property.

5.1 Increasing Returns Functions

Consider the merger game outlined in section 2, where firms know their own z_i, but not their competitor's type $z_j, j \neq i$. However, suppose that firm types are relation-specific in the sense that they do not affect the firms' profits in the absence of the transaction, implying that $\pi_i, i = 1, 2$, is independent of (z_i, z_j). However, if the transaction occurs, the joint profit $\pi^M (z_i, z_j)$ is monotone increasing in both z_i and z_j. A plausible example of such a setting arises when the two firms are providers of complementary assets that are essential to carry out particular projects, but not valuable outside the relationship. Then, provided that each party receives a positive share of each

[21] More specifically, for $\alpha = 0.5$, there is a non-trivial cut-off equilibrium for high uncertainty ($\gamma = 20$). For small and medium uncertainty ($\gamma = 2, 5$), all types consent to the merger.

increase in π^M, the returns from the transaction, $g_i(z_i, z_j) = \pi_i^M(z_i, z_j) - \pi_i$, are monotone increasing rather than decreasing in own state.

Intuition suggests that in such a setting, only high types consent to the transaction. Contrary to the assumptions of Proposition 1, g_i is monotone increasing in own type, so that high types gain more than low types in expectation, and the equilibria will be of the opposite cut-off type, i.e. only good firms will consent to the merger.

5.2 Non-Monotone Returns Functions

As mentioned above, demanding that merger returns are monotone decreasing in own type is more restrictive than necessary to derive a cut-off equilibrium: By Lemma 1, it is sufficient that expected merger returns satisfy SSC^-. In the following, we show that a large class of functions satisfies this condition.

To apply Proposition 1, we need to understand under which conditions the merger return function satisfies downward single-crossing in expectation. To this end, we define the class of *essentially monotone decreasing functions* as follows. Suppose $h : \mathcal{X} \times \mathcal{Y} \longrightarrow \mathbb{R}$ is an arbitrary function on $\mathcal{X} \times \mathcal{Y} \subset \mathbb{R}^2$, and let μ denote the standard Lebesgue measure on $\mathbb{R}$. We use the following notation:

Notation 1

- $A(x) \equiv \{y \in \mathcal{Y} \mid h(x, y) \geq 0\}$
- $D(x) \equiv \{y \in \mathcal{Y} \mid h(x, y) \leq 0\}$
- $C \equiv \{x \in \mathcal{X} \mid \mu(A(x)) > 0 \wedge \mu(D(x)) > 0\}.$

Our merger model (where $x = z_i, y = z_j$ and $h = g_i$) serves to motivate the notation. $A(z_i)$ corresponds to the set of types z_j for which firm i with state z_i accepts an agreement under complete information, and $D(z_i)$ corresponds to the set of types z_j for which it declines.[22] C is the "critical" set of types

[22] In both cases, the types z_j for which firm i with state z_i is indifferent are also included.

z_i who accept an agreement with some types z_j, but decline it with others. The following definition is useful.

Definition 3 *(a) h is* ***essentially monotone decreasing*** *(EMD) in x if the following properties are satisfied:*

(EMD1) $\forall x^2 \geq x^1 : \mu\left(A\left(x^1\right)\right) = 0 \Longrightarrow \mu\left(A\left(x^2\right)\right) = 0.$

(EMD2) $\forall x^1 \leq x^2 : \mu\left(D\left(x^2\right)\right) = 0 \Longrightarrow \mu\left(D\left(x^1\right)\right) = 0.$

(EMD3) $\forall x^1, x^2 \in C, x^1 < x^2 :$

$$\begin{aligned} &h\left(x^1, y\right) > h\left(x^2, y\right) \text{ for } \mu\text{-almost all } y \in A\left(x^1\right) \text{ or } A\left(x^2\right), \qquad \text{or} \\ &h\left(x^1, y\right) > h\left(x^2, y\right) \text{ for } \mu\text{-almost all } y \in D\left(x^1\right) \text{ or } D\left(x^2\right). \end{aligned}$$

(EMD4) *The restriction of h to the subset $C \subset \mathcal{X}$, $h|_C(x, y)$, is non-increasing in x for μ-almost all $y \in \mathcal{Y}$.*

h is ***essentially monotone decreasing in x in the weak sense*** *(WEMD) if (EMD1)–(EMD3) hold.*

(b) h is ***essentially monotone increasing*** *(EMI) in x if $-h$ is EMD in x. h is* ***essentially monotone increasing in x in the weak sense*** *(WEMI) if $-h$ is WEMD in x.*

Consider properties (EMD1)-(EMD4). In the model of section 2, the interpretation of (EMD1) is that, when some low type z_i does not gain from mergers with a positive measure of other types, then neither does any higher type $z_i' > z_i$. The interpretation of (EMD2) is similar: When some high type z_i' will enter an agreement with almost every type z_j, then any lower type $z_i < z_i'$ will do so, too. (EMD3) and (EMD4) are additional monotonicity requirements on the critical set of types who consent for some types, but not for others.

To understand which functions satisfy EMD, it is useful to summarize results on the relation between EMD and more familiar concepts.

Lemma 2 *(i) $h(x, y)$ is EMD in x if it satisfies one of the following properties $(i.1)$-$(i.4)$:*

(i.1) $h(x,y)$ *is monotone decreasing in* x *for* μ*-almost all* $y \in \mathcal{Y}$.

(i.2) $h(x,y) > 0$ *for all* $x \in \mathcal{X}$ *and* μ*-almost all* $y \in \mathcal{Y}$.

(i.3) $h(x,y) < 0$ *for all* $x \in \mathcal{X}$ *and* μ*-almost all* $y \in \mathcal{Y}$.

(i.4) $\underline{x} = \min \mathcal{X}$ *exists;* $h(x,y)$ *is single-peaked in* x *with peak* $x^*(y)$ *and* $\widehat{x} = \sup_{y \in \mathcal{Y}} x^*(y)$ *such that* $\widehat{x} \leq \widetilde{x} \equiv \inf C$.

(ii) *If* h *is EMD in* x*, then* $h(x,y)$ *satisfies* SSC^- *in* x *for* μ*-almost all* $y \in Y$*, i.e., for all* $x^H, x^L \in X$ *such that* $x^H > x^L, h(x^H) \geq 0$ *implies* $h(x^L) \geq 0$ *and* $h(x^H) > 0$ *implies* $h(x^L) > 0$

Proof. See Appendix. ■

Part (i) of Lemma 2 states that EMD contains monotone decreasing functions (1), functions that have the same sign independent of x ((2) and (3)), and a large class of functions that are single-peaked in x (4). Figure 3 gives an example for the latter case: Beyond single-peakedness, EMD requires that the peaks of the function are to the left of the critical set $[\underline{c}, \overline{c}]$.

Part (ii) states that EMD implies strong downward single crossing. As Figure 4 shows, the converse statement does not hold, because EMD includes monotonicity properties for the critical set $C \equiv [\underline{c}, \overline{c}]$, whereas SSC^- does not. Even WEMD does not hold in this particular example.

<Figures 3 and 4 around here>

The next result is crucial to show that EMD is sufficient for the cut-off result that low types are more likely to engage in mergers than high types.

Proposition 4 *If* h *satisfies (EMD), then it also satisfies* ***downward single-crossing in expectation*** *(*ESC^-*), i.e.* $\int_B h(x,y) f(y)\, dy$ *satisfies* SSC^- *in* x *for every pair* (B, f) *where* B *is a subset of* $\mathbb{R}$ *and* $f : B \rightarrow \mathbb{R}_+$ *is a function with* $\int_B f(y)\, dy > 0$.

Proof. See Appendix. ■

Together with Lemma 1, Proposition 4 implies that a cut-off equilibrium exists where only low types consent to the merger, provided that g_i is EMD.[23,24]

Finally, note that EMD is not necessary for ESC^-. Nevertheless, we can provide a partial converse of Proposition 4.

Proposition 5 *If ESC^- holds, h satisfies WEMD.*

Proof. See Appendix ■

6 Conclusions

Mergers under asymmetric information differ from familiar lemons problems in two ways. First, the informational asymmetry is two-sided. Second, high types earn more both before and after a transaction takes place. Thus, it is not obvious that merger returns are monotone decreasing in own type and that low types are more likely to consent to transactions than high types. In spite of this ambiguity, we show that, in a Cournot setting with linear demand, there are cases where merger returns are monotone decreasing in own type, so that standard lemons results hold. Beyond that, we give a more general condition on merger returns—that they are "essentially monotone decreasing"—guaranteeing that low types are more likely to merge than high types.

We also show that our merger game always has a no-merger equilibrium, and we give conditions for this equilibrium to be unique.

Applying the general findings in the Cournot setting yields several interesting results. For instance, when uncertainty is large, equilibria where

[23] An alternative sufficient condition for ESC^- could have been obtained by applying the same logic as in Athey (2000, Theorem 5). See Borek et al. (2003) for further details.

[24] If h satisfies EMI (rather than EMD), the result is reversed: h then satisfies upwards single-crossing in expectation, so that in the Bayesian equilibrium only high types consent to the merger.

mergers occur may exist, whereas no such equilibrium would exist under symmetric information. Intuitively, when there is considerable uncertainty about firm types, the potential partners to a merger can hope that they will turn out to be sufficiently asymmetric ex post for substantial rationalization effects to materialize. However, it depends on the profit sharing rule whether this is possible: When firms commit to a fixed share of profits ex ante, mergers will never occur in the absence of synergies, whereas they may occur when firms commit to a fixed price.

Appendix

Proof of Lemma 1

Firm i's expected merger return, facing firm j with strategy s_j, is

$$\begin{aligned} G_i(z_i; B_j, f_j) = {} & \mathbb{P}[B_j]\, \mathbb{E}_{z_j}\left[\pi_i^M(z_i, z_j)\, |z_j \in B_j(s_j)\right] + \\ & (1 - \mathbb{P}[B_j])\, \mathbb{E}_{z_j}\left[\pi_i(z_i, z_j)\, |z_j \notin B_j(s_j)\right] - \mathbb{E}_{z_j}\left[\pi_i(z_i, z_j)\right] \\ = {} & \int_{B_j} g_i(z_i, z_j)\, f_j(z_j)\, dz_j. \end{aligned}$$

If $G_i(z_i; B_j, f_j)$ is positive, firm i will consent to the merger, otherwise it will reject the merger. By assumption, $\int_{B_j} g_i(z_i, z_j)\, f_j(z_j)\, dz_j$ satisfies SSC^- in z_i. Denote the single crossing points required by Definition 2 as $z_i^\circ(s_j)$. Now define

$$\tilde{R}_i(s_j) = \begin{cases} z_i^\circ(s_j), & \text{if } z_i^\circ(s_j) \le \overline{z_i}; \\ \overline{z_i}, & \text{if } z_i^\circ(s_j) \ge \overline{z_i} \text{ or if } z_i^\circ(s_j) \text{ does not exist.} \end{cases}$$

Then firm i's optimal reaction is

$$R_i(z_i, s_j) = \begin{cases} 1, & \text{if } z_i \le \tilde{R}_i(s_j); \\ 0, & \text{if } z_i > \tilde{R}_i(s_j). \end{cases}$$

In particular, for an equilibrium strategy s_j, the best reply has the required cut-off structure.

Proof of Proposition 2

Suppose firm i plays strategy $s_i(z_i)$ with $\mathbb{P}[B_i(s_i)] = 0$. Then the probability that a merger takes place is zero and therefore firm $j \neq i$ is indifferent between any strategies it can play; in particular, every strategy $s_j(z_j)$ with $\mathbb{P}[B_j(s_j)] = 0$ is a best response.

Proof of Proposition 3

(i) Assume that, for all $i \in \{1, 2\}$, there is no $\widehat{z}$ such that $g_i(\widehat{z}_i, \widehat{z}_i) \geq 0$. Suppose w.l.o.g. that there is a non-trivial cut-off equilibrium (z_1^*, z_2^*) with $z_1^* \geq z_2^*$. As $g_1(z_1^*, z_1^*) < 0$ and g_1 is monotone increasing in z_2, $g_1(z_1^*, z_2) < 0$ for all $z_2 \leq z_2^*$. Therefore, expected equilibrium profits for firm 1 are $\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2) dz_2 < 0$, contradicting the condition that $\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2) dz_2 = 0$ for the cut-off values (z_1^*, z_2^*).

(ii) Suppose $z_1^* = z_2^* \equiv z^*$ and assume w.l.o.g. that $g_1(z^*, z^*) < 0$. Then $g_1(z^*, z_2) < 0$ for all $z_2 < z^*$. Thus expected equilibrium profits for firm 1 are $\int_{\underline{z}_2}^{z^*} g_1(z^*, z_2) f_2(z_2) dz_2 < 0$, contradicting the condition that $\int_{\underline{z}_2}^{z^*} g_1(z^*, z_2) f_2(z_2) dz_2 = 0$ for the cut-off values (z^*, z^*).

Proof of Lemma 2

(i.1) To show (EMD1) let $x^1 \in \mathcal{X}$ such that $\mu(A(x^1)) = 0$. Then $h(x^1, y) < 0$ for μ-almost all $y \in \mathcal{Y}$. Since $h(x, y)$ is monotone decreasing in x for μ-almost all $y \in \mathcal{Y}$ it follows that $h(x^2, y) < h(x^1, y) < 0$ for all $x^2 > x^1$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(A(x^2)) = 0$ for all $x^2 \geq x^1$. The proof of (EMD2) is analogous, (EMD3) and (EMD4) are obvious.

(i.2) If $h(x, y) > 0$ for all $x \in \mathcal{X}$ and μ-almost all $y \in \mathcal{Y}$, then $\mu(A(x)) > 0$, $\mu(D(x)) = 0$ for all $x \in \mathcal{X}$, and $C = \emptyset$. Therefore (EMD1)–(EMD4) are trivially satisfied.

(i.3) An analogous argument holds for $h(x, y) < 0$.

(i.4) (EMD1) Let $x^1 \in \mathcal{X}$ such that $\mu(A(x^1)) = 0$. Then $h(x^1, y) < 0$ for μ-almost all $y \in \mathcal{Y}$. Since $\underline{h}(y) > 0 > h(x^1, y)$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^2, y) < h(x^1, y) < 0$ for all $x^2 > x^1$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(A(x^2)) = 0$ for all $x^2 \geq x^1$.

(EMD2) Let $x^2 \in \mathcal{X}$ such that $\mu(D(x^2)) = 0$. Then $h(x^2, y) > 0$ for μ-almost all $y \in \mathcal{Y}$. Since $\underline{h}(y) > 0$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^1, y) > 0$ for all $x^1 \leq x^2$ and μ-almost all $y \in \mathcal{Y}$. Therefore $\mu(D(x^1)) = 0$ for all $x^1 \leq x^2$.

(EMD3) Let $x^1, x^2 \in C,\ x^1 < x^2$, and $y \in D(x^1)$. Since $\underline{h}(y) > 0 \geq h(x^1, y)$ and $h(x, y)$ is single-peaked in x for μ-almost all $y \in \mathcal{Y}$, it follows that $h(x^1, y) > h(x^2, y)$ for μ-almost all $y \in D(x^1)$.

(EMD4) As $h(x, y)$ is single-peaked in x, it is non-increasing in x for $x \geq \widehat{x}$ and μ-almost all $y \in \mathcal{Y}$. As $\widetilde{x} \geq \widehat{x}$, $h(x, y)$ is non-increasing in x for $x \in C$ and μ-almost all $y \in \mathcal{Y}$.

(ii) Suppose $h(x, y)$ does not satisfy SSC$^-$ in x for μ-almost all $y \in \mathcal{Y}$. Then there exists $x^1 < x^2$ and $M \subset \mathcal{Y}$ with $\mu(M) > 0$ such that $h(x^1, y) \leq 0 \leq h(x^2, y)$ for all $y \in M$. This implies $\mu(A(x^2)) > 0$ as $M \subset A(x^2)$. Since $x^1 < x^2$ we get $\mu(A(x^1)) > 0$ by (EMD1). Thus $x^1 \in C$. An analogous argument shows that $x^2 \in C$. Now if $h(x^1, y) < h(x^2, y)$ for μ-almost all $y \in M$, we have a contradiction to (EMD4), and if $h(x^1, y) = 0 = h(x^2, y)$ for μ-almost all $y \in M$, we have a contradiction to (EMD3).

Proof of Proposition 4

(a) Let $x \in \mathcal{X}$ such that $\mu(D(x)) = 0$. Then $h(x, y) > 0$ for μ-almost all y. Therefore $\int_B h(x, y) f(y)\, dy > 0$ for all (B, f) with $\int_B f(y)\, dy > 0$.

(b) For $x \in \mathcal{X}$ with $\mu(A(x)) = 0$ an analogous argument shows that $\int_B h(x, y) f(y)\, dy < 0$ for all (B, f) with $\int_B f(y)\, dy > 0$.

(c) Let $x^1, x^2 \in C$ such that $x^1 < x^2$. From (EMD4) we know that $\int_B h(x^1, y) f(y) \, dy \geq \int_B h(x^2, y) f(y) \, dy$ for all (B, f) with $\int_B f(y) \, dy > 0$. Now suppose that there exists (B_0, f_0) with $\int_{B_0} f_0(y) \, dy > 0$ such that

$$\int_{B_0} h\left(x^1, y\right) f_0(y) \, dy = 0 = \int_{B_0} h\left(x^2, y\right) f_0(y) \, dy. \tag{2}$$

This implies $\mu(A(x^i) \cap B_0) > 0$ and $\mu(D(x^i) \cap B_0) > 0$ for $i = 1, 2$. Together with (EMD3) and (EMD4) we get $\int_{B_0} h(x^1, y) f_0(y) \, dy > \int_{B_0} h(x^2, y) f_0(y) \, dy$, which is a contradiction to (2). Thus, for h restricted to C, $\int_{B_0} h(x, y) f(y) \, dy$ satisfies single crossing.

(d) (EMD1) and (EMD2) guarantee that $\sup\{x \in \mathcal{X} \mid \mu(D(x)) = 0\} = \inf C$ and $\sup C = \inf\{x \in \mathcal{X} \mid \mu(A(x)) = 0\}$, and so (a)–(c) establish SSC$^-$ for $\int_B h(x, y) f(y) \, dy$ for all (B, f) with $\int_B f(y) \, dy > 0$.

Proof of Proposition 5

(a) Suppose that (EMD1) is not satisfied. Then there exists $x^1 < x^2$ such that $\mu(A(x^1)) = 0$ and $\mu(A(x^2)) > 0$. Therefore we can find $B \subset A(x^2)$ and f such that $\int_B f(y) \, dy > 0$. We thus get $\int_B h(x^1, y) f(y) \, dy < 0 \leq \int_B h(x^2, y) f(y) \, dy$, which is a contradiction to SSC$^-$.

(b) The necessity of (EMD2) is proven analogously.

(c) Suppose (EMD3) is not satisfied. Then there exists $x^1, x^2 \in C, x^1 < x^2$ and $M_1 \subset A(x^1), M_2 \subset D(x^1)$ with $\mu(M_i) > 0$, $i = 1, 2$, such that $h(x^1, y) \leq h(x^2, y)$ for all $y \in M_1 \cup M_2$. Therefore we can find $(B \subset M_1 \cup M_2, f)$ with $\int_B f(y) \, dy > 0$ and such that $0 = \int_B h(x^1, y) f(y) \, dy \leq \int_B h(x^2, y) f(y) \, dy$, which contradicts SSC$^-$. This completes the proof.

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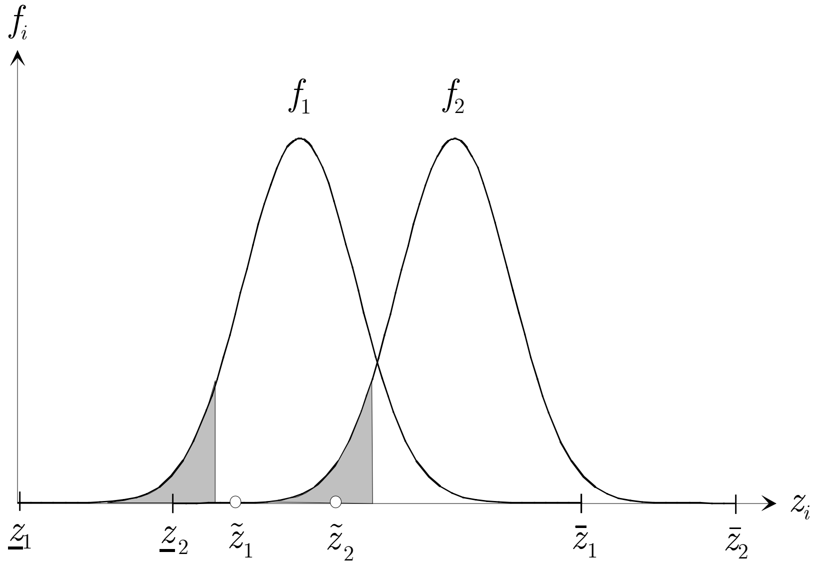

Figure 1: Ex ante heterogeneity of firms

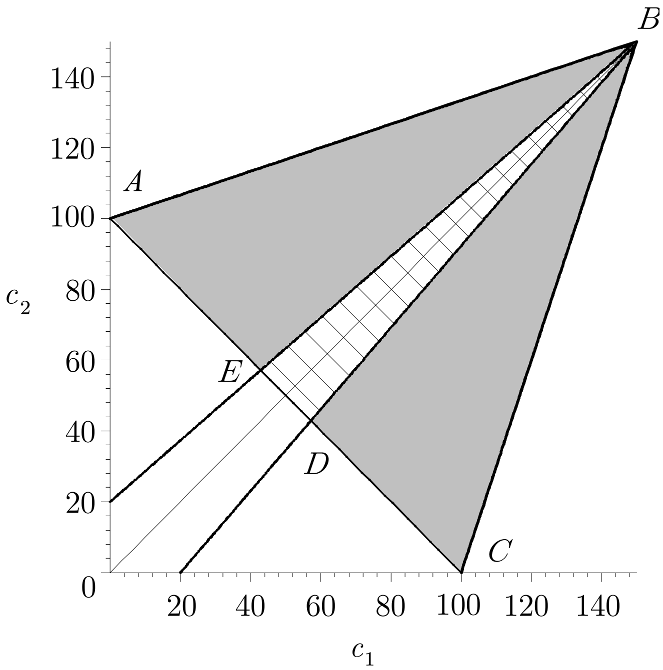

Figure 2: Admissible range of marginal costs ($a = 2, b = 1, c_3 = 1$).

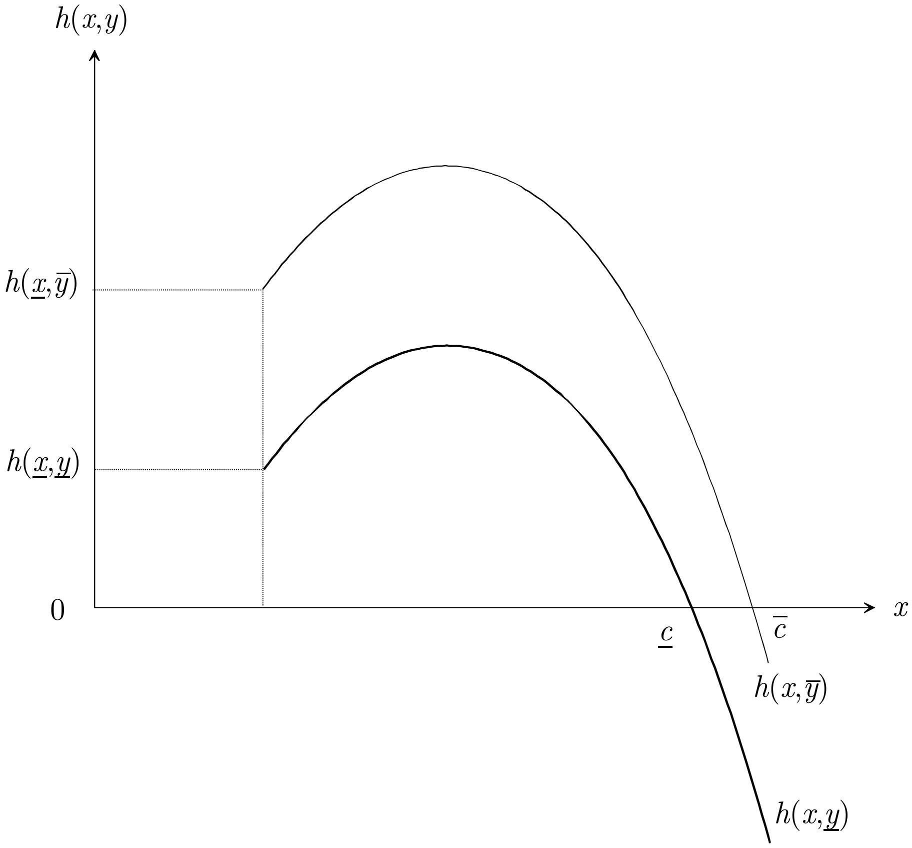

Figure 3: Single-peaked functions $h(x, y)$

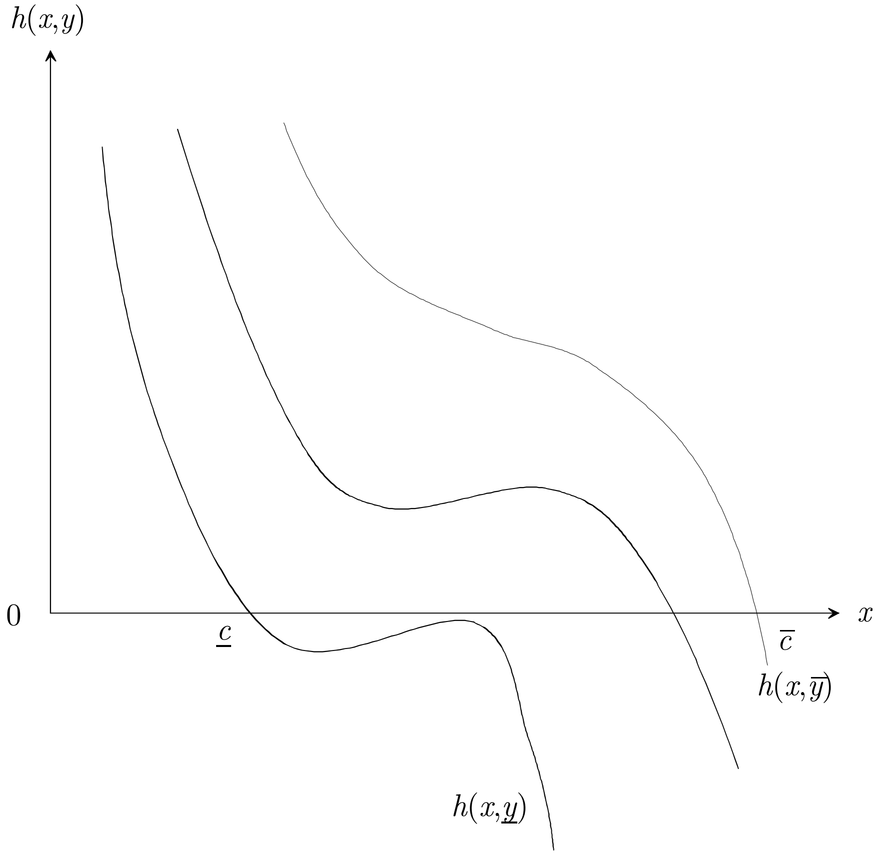

Figure 4: $h(x, y)$ is SSC^{-} but not WEMD

Table 1: Merger Pattern in the Cournot model

	Rationalization Merger $z^m = \max(z_1, z_2)$		
Profit Sharing	Uncertainty		
	High ($\gamma = 20$)	Medium ($\gamma = 5$)	Low ($\gamma = 2$)
Fixed Share	$\emptyset$	$\emptyset$	$\emptyset$
Joint Surplus	all types	$\emptyset^*$	$\emptyset$
Cash Payment	low types	$\emptyset$	$\emptyset$

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